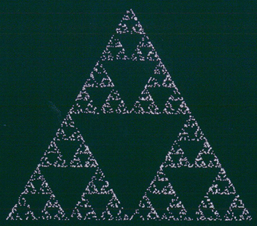
FRACTALS

PROJECT GOALS

studying properties of the Sierpinski triangle and the Koch snowflake

calculating fractal dimensions of Croatian and Cyprus coastline

The Sierpinski triangle



The Sierpinski triangle made by using Chaos game

$P_n = 2a\left(\frac{3}{2}\right)^n$ $A_n = \frac{a^2\sqrt{3}}{3}\left(\frac{3}{4}\right)^n$

a – side length
n – step of iteration

$\frac{3}{2} > 1$: $\left(\frac{3}{2}\right)^n$ gets larger as $n \to \infty$ $\frac{3}{4} < 1$: $\left(\frac{3}{4}\right)^n$ gets smaller as $n \to \infty$

Perimeter tends to infinity while area tends to zero.

Fractal dimension

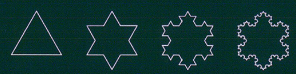
$$D = log_k n$$

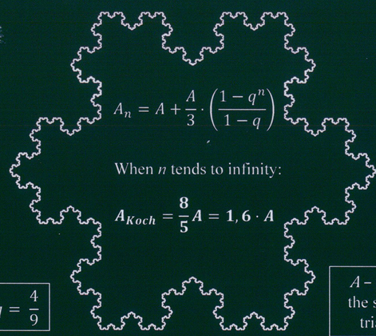
D – fractal dimension
k – magnification factor
n – number of self-similar segments

$D_{Sierpinski} = log_2 3$ $D_{Koch} = log_3 4$

$D_{Sierpinski} \approx 1{,}585$ $D_{Koch} \approx 1{,}262$

The Koch snowflake





$A_n = A + \frac{A}{3} \cdot \left(\frac{1-q^n}{1-q}\right)$

When n tends to infinity:

$A_{Koch} = \frac{8}{5}A = 1{,}6 \cdot A$

$q = \frac{4}{9}$

A – area of the starting triangle

$P = \left(\frac{4}{3}\right)^{n-1}$ $\frac{4}{3} > 1$: $\left(\frac{4}{3}\right)^{n-1}$ gets larger as $n \to \infty$

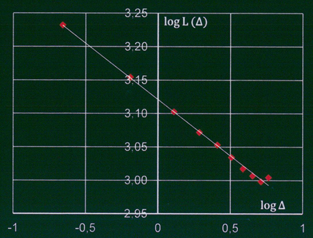
Perimeter tends to infinity while area stops at $1{,}6 \cdot A$.

The Coastline paradox

Length of the coastline depends on the length of the measuring stick.

CROATIA

N	L (Δ) / km	Δ / km
1	1041	3,87
2	1130	2,58
3	1267	1,29
4	1427	0,65
5	1706	0,22

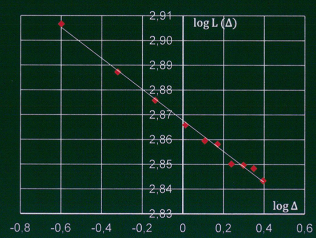


$y = -0{,}168x + 3{,}121$

$D = 1{,}168$

CYPRUS

N	L (Δ) / km	Δ / km
1	697	2,49
2	708	1,98
3	734	1,03
4	771	0,48
5	807	0,25



$y = -0{,}063x + 2{,}868$

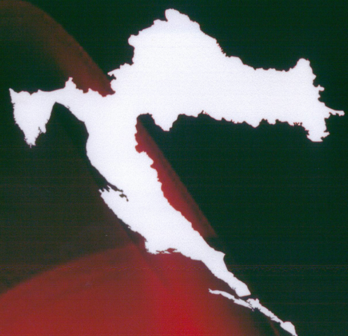
$D = 1{,}063$

β – slope of the line
D – fractal dimension
L – length of the coastline
Δ – length of the measuring stick

$L_0 = C\Delta_0^{1-D}$ $L = C\Delta^{1-D}$

$$log\frac{L_0}{L} = log\left(\frac{\Delta_0}{\Delta}\right)^{1-D}$$

$$D = 1 - \beta$$



Conclusion:

The Croatian coastline is more jagged than the one of Cyprus because its fractal dimension is bigger.

1,168 > 1,063

21